ADDITIONAL MATHS: CALCULUS ASSIGNMENT

1. The velocity, $v\,\mathrm{m\,s^{-1}}$, of a particle, travelling in a straight line, at time t s after leaving a fixed point O, is given by

$$v = 10 + kt - 3t^2,$$

where $t \geqslant 0$ and k is a constant. When $t = 0$ the particle is at O and its acceleration is $1\,\mathrm{m\,s^{-2}}$. Find

(i) the value of k,

(ii) the value of t when the particle is instantaneously at rest,

(iii) the distance the particle has travelled when it is again at O.

[7]

2. A closed can, in the shape of a circular cylinder, is to contain $500\,\mathrm{cm^3}$ of liquid when full. The cylinder, of radius r cm and height h cm, is made from thin sheet metal. The total external surface area of the cylinder is $A\,\mathrm{cm^2}$.

(i) Show that $A = 2\pi r^2 + \dfrac{1000}{r}$.

(ii) Find the value, to two significant figures, of r and of h for which A has a stationary value.

(iii) Calculate the stationary value of A and determine whether it is a maximum or a minimum.

[12]

3. (a) Integrate $\displaystyle\int\left(6x^3 - \frac{5}{x^6} + 9\sqrt{x}\right)\mathrm{d}x$ [3]

(b) Evaluate, using integration, (i) $\displaystyle\int_{-1}^{3}(x+1)(x-3)\,\mathrm{d}x$ [3]

(ii) $\displaystyle\int_{1}^{2}\frac{3x^4 - 4x^2 + 6x^5}{2x^4}\,\mathrm{d}x$ [4]

4.

(i) Verify that the point P, whose coordinates are $(2p, p^2)$, lies on the curve $4y = x^2$. [1]

(ii) By finding the gradient of the curve at P in terms of p, prove that the equation of the tangent to the curve at P is $y - px + p^2 = 0$. [4]

(iii) Q is the point on the curve whose coordinates are $(2q, q^2)$. Write down the equation of the tangent to the curve at Q in terms of q. [2]

(iv) If the tangents at P and Q are perpendicular, state the value of the product pq. [1]

5. A particle of mass 3 kg moves so that at time t seconds its position is given by the vector

$$\mathbf{r} = \begin{pmatrix} t^2 - 4t - 12 \\ t^2 - 4t - 5 \end{pmatrix},$$

where the units are in metres.

(i) Find an expression for the velocity vector $\mathbf{v}$ in terms of t.

Deduce the speed of the particle after 5 seconds and state the value of t for which the particle is at rest. [4]

(ii) What is the velocity at the point $\mathbf{r} = \begin{pmatrix} 20 \\ 27 \end{pmatrix}$? [3]

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6. **(a)** Differentiate with respect to x

(i) $y = 7x^{1.2} + 4x^{-3}$, [2]

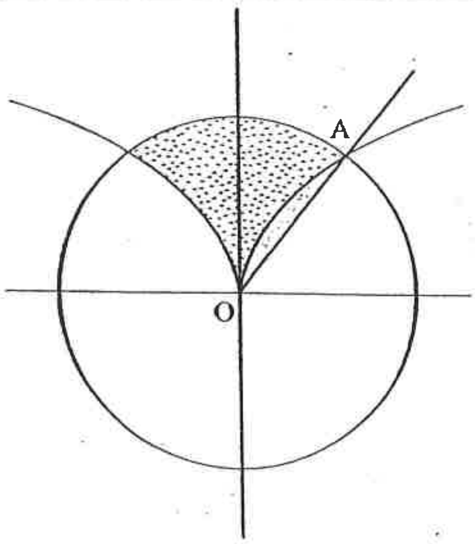
(ii) $y = x(x^3 - 2)^2$, [4]

(iii) $y = \dfrac{3x^5 - 4x^2}{2x^4}$. [3]

(b) $y = \dfrac{x^3}{3} - ax^2 + bx$ has turning values when $x = 3$ and $x = 5$.

Find the values of a and b. [5]

7.



The figure shows the graphs of $x^2 + y^2 = 32$, $4x^2 = y^3$ and $y = x$. The point A lies on all three graphs.

(i) Calculate the coordinates of the point A. [2]

(ii) Calculate the area of the shaded region bounded by the straight line OA and the curve OA. [6]

(iii) Deduce the area of the dotted region bounded by the circle and the two branches of the graph $4x^2 = y^3$. [3]

(iv) Calculate the gradient of the tangent to the curve $4x^2 = y^3$ at the point (4, 4). [3]

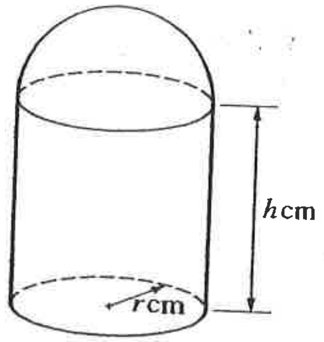
(i) Find the equation of the circle A with centre $(2, -4)$ and radius 4. [3]

CALCULUS II

8. Find the gradient of the curve $y = 4x^2 - 20x + 27$ at the point $P(2, 3)$. [2]

The tangent to the curve at P meets the x-axis at A. Calculate the area of the triangle AOP, where O is the origin. [3]

9.



[A sphere of radius r has a volume of $\frac{4}{3}\pi r^3$ and a surface area of $4\pi r^2$.]
The diagram shows a solid body which consists of a hemisphere fixed, with no overlap, to the end of a right circular cylinder of radius r cm and height h cm. Given that the total volume of the solid is $360\pi\,\text{cm}^3$, express h in terms of r.

Hence show that the total surface area, $A\,\text{cm}^2$, of the solid is given by

$$A = 5\pi\left(\frac{144}{r} + \frac{r^2}{3}\right)$$

[4]

Given that r can vary, find

(i) the value of r for which A has a stationary value, [4]

(ii) the overall height of the body when A has a stationary value, [1]

(iii) the stationary value of A. [1]

Determine whether the stationary value of A is a maximum or a minimum. [2]

0.

A particle P moves in a straight line so that, t seconds after passing through a fixed point O, its velocity $v\,\text{cms}^{-1}$ is given by $v = 3t^2 - 15t + 18$. Find

(i) the value of t when the velocity of P is equal to its initial velocity, [2]

(ii) the values of t for which P is instantaneously at rest, [2]

(iii) an expression, in terms of t, for the distance of P from O at time t, [2]

(iv) the total distance travelled by P in the first 4 seconds after passing through O, [3]

(v) the distance of P from O when the acceleration of P is zero. [3]

1 A curve passes through the point $P(0, \frac{7}{2})$ and is such that $\frac{dy}{dx} = 2 - x$. The normal to the curve at P meets the curve again at Q. Find

(i) the coordinates of Q,

(ii) the area of the region between the curve and the line PQ.

[12]

Show that the tangent to the curve

$$y = \frac{3}{x} - \frac{4}{x^2}$$

at the point $(2, \frac{1}{2})$ passes through the origin.

[4]

13. A curve is such that $\frac{dy}{dx} = 5 - 3x$. The line $y = x + 2$ meets the curve at the point P, where the gradient of the curve is 2. Find

(i) the coordinates of P,

(ii) the equation of the curve.

[5]

14. A particle moves in a straight line so that, t seconds after leaving a fixed point O, its velocity, $v\,\text{ms}^{-1}$, is given by $v = t^3 - 3t^2 + 2t$. Find

(i) the acceleration of the particle when $t = 1.5$,

(ii) the values of t when the particle is at instantaneous rest,

(iii) the distance travelled in the interval $0 \leqslant t \leqslant 2$.

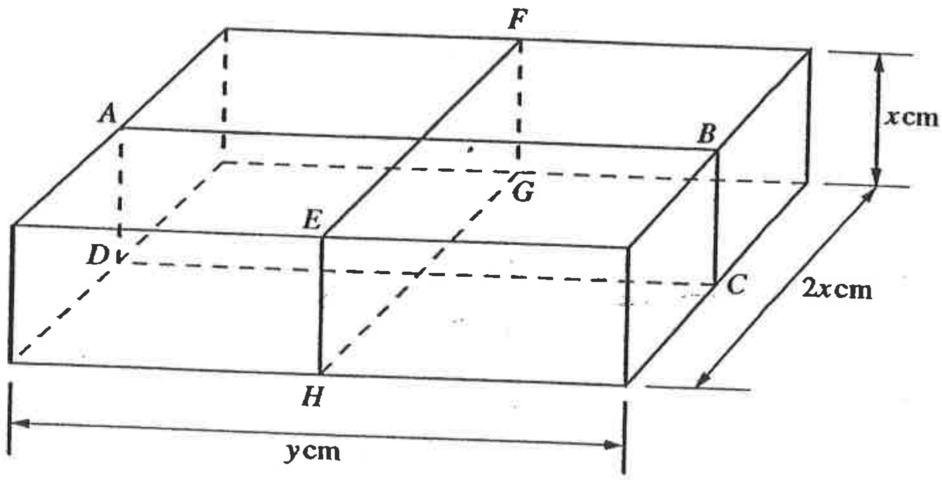
[7]

15.

(a) The two variables, x and y, are related by the equation $x + y = 3$. Find the maximum value of the variable z, where $z = 5x^3y$.

[5]

(b)



The diagram shows a package in the shape of a rectangular block whose sides are of length x cm, $2x$ cm and y cm. The package is secured by two pieces of string, $ABCDA$ and $EFGHE$, whose total length is 300 cm. The volume of the package is $V\,\text{cm}^3$.

(i) Show that $V = 300x^2 - 8x^3$.

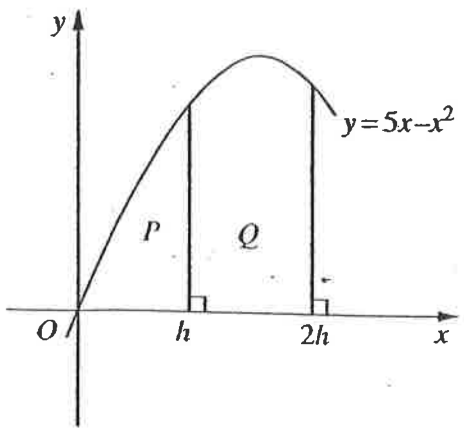
Given that x varies,

(ii) find the value of x for which V is stationary and determine the nature of the stationary value.

[7]

CALCULUS III

16



The region P is bounded by the curve $y = 5x - x^2$, the x-axis and the line $x = h$. The region Q is bounded by the curve $y = 5x - x^2$, the x-axis and the lines $x = h$, $x = 2h$. Given that the area of Q is twice the area of P, find the value of h. [6]

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17 **(a)** Differentiate with respect to x

(i) $y = 2x^5 - x^3 + 3x^2 - 5$, [2]

(ii) $y = \dfrac{2}{\sqrt{x}}$, [2]

(iii) $y = 2x^3\left(3x^4 - \dfrac{4}{x^4}\right)$. [3]

17 **(b)** The curve with equation $y = x^3 - 9x^2 + ax + b$ has a turning point at $(1, 3)$.

(i) Find the values of a and b. [5]

(ii) Determine whether the turning point at $(1, 3)$ is a maximum or a minimum. [2]

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8 **(a)** Find the integral $\displaystyle\int \left(2x + \frac{1}{x}\right)^2 \mathrm{d}x$. [4]

(b) Evaluate $\displaystyle\int_1^4 \frac{4x^5 - 3x^{\frac{5}{2}}}{2x^2}\,\mathrm{d}x$. [5]

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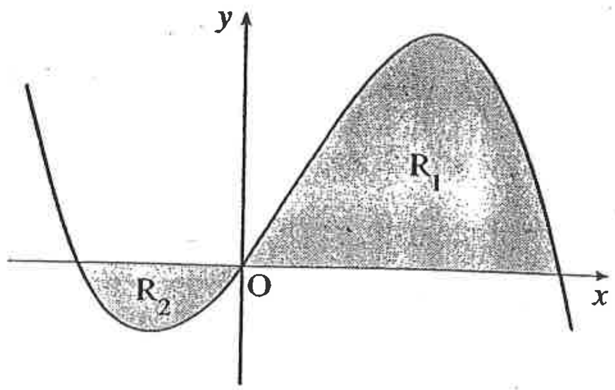
9 **(i)** The point P on the curve $y = \dfrac{10}{x}$ has x-coordinate 2. Find the equation of the tangent to the curve at P. [4]

(ii) The point Q on the curve $y = -x^2 + 6x - 3$ has x-coordinate 5. Find the equation of the normal to the curve at Q. [4]

(iii) The curves $y = \dfrac{10}{x}$ and $y = -x^2 + 6x - 3$ intersect at P and Q, and at another point R. Write down and simplify an equation satisfied by the x-coordinates of P, Q and R. Hence find the coordinates of R. [6]

20.

(a)

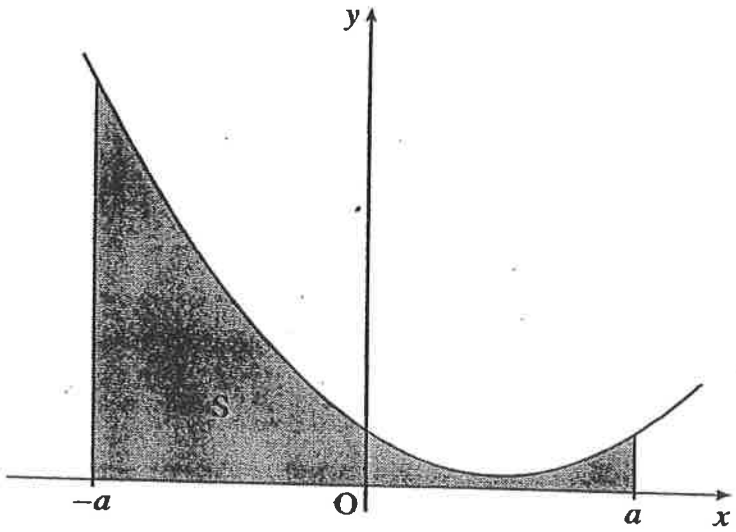


The diagram shows the graph of $y = x(2-x)(1+x)$. Find

(i) the area of the region R_1, [5]

(ii) the total area of the regions R_1 and R_2. [3]

(b)



The region S is bounded by the x-axis, the curve $y = 3x^2 - 6x + 4$ and the lines $x = -a$ and $x = a$, where $a > 0$.

(i) Find the area of S in terms of a. [3]

(ii) Given that the area of S is 32 units^2, show that $a^3 + 4a - 16 = 0$. Find the value of a given that it is an integer. [3]