ADDITIONAL MATHS: COORDINATE GEOMETRY ASSIGNM (including some Calculus)

1.

A is the point (2, 5) and the line joining the points A and B has a gradient of $\frac{1}{3}$. The perpendicular bisector of AB passes through the point (4, 9). Find

(i) the equation of AB,

(ii) the coordinates of B.

[6]

2.

Find the value of the constant c for which the line $3y = x + c$ is a normal to the curve $y = x^2 - x + 3$.

[5]

3.

(a) The gradient at any point (x, y) on a curve is $3 + \frac{1}{x^3}$. At the point on the curve where $y = \frac{1}{2}$ the gradient is 2. Find

(i) the equation of the curve,

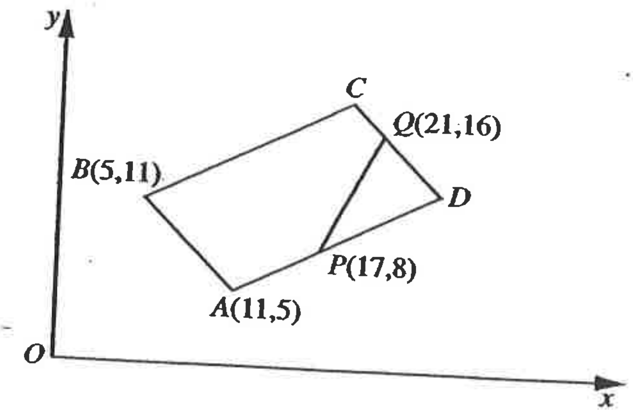
(ii) the equation of the tangent to the curve at the point on the curve where $x = 1$.

[7]

(b) The equation of a curve is $y = 3x^2 - kx + 2$, where k is a constant. The tangent to the curve, at the point where $x = 2$, passes through (5, 5). Find the value of k.

[5]

4. **Solutions to this question by accurate drawing will not be accepted.**



In the diagram the points A (11, 5), B (5, 11), C and D are the vertices of a parallelogram. The points P (17, 8) and Q (21, 16) lie on AD and CD respectively.

(i) Find the equation of AD and of CD.

(ii) Find the ratio $DQ : QC$.

(iii) Show that triangle PDQ is isosceles and determine its area.

[12]

5.

(i) C is the centre of the circle whose equation is $x^2 + y^2 - 24x - 10y + 144 = 0$.
Find the coordinates of C and the radius of the circle. [3]

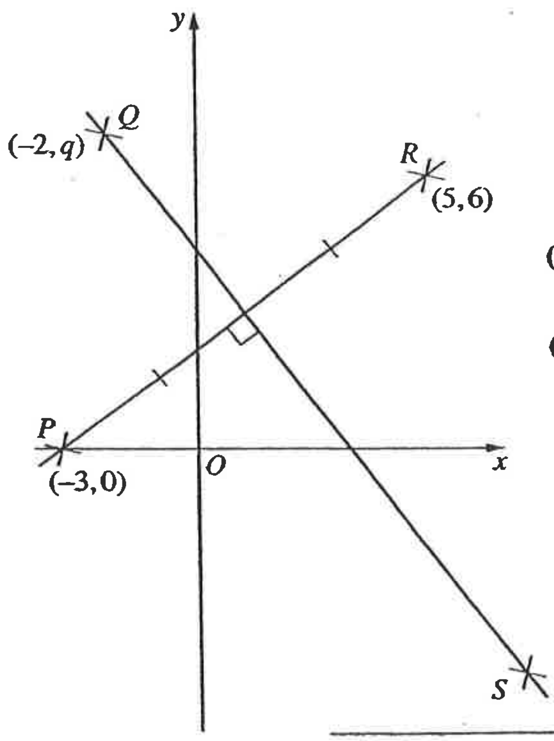
(ii) Find the distance of C from the origin O and deduce the equations of the two circles whose centre is O and which touch the original circle. [3]

(iii) Q is the point of contact of the original circle and the smaller of the two circles centre O. Calculate the coordinates of Q and the equation of the common tangent to the two circles at Q. [4]

(iv) Verify that this common tangent passes through the point P(12, −8). Hence, by noting that PC is vertical, obtain the equation of the other tangent to the original circle that can be drawn through P. [4]

6

Solutions to this question by accurate drawing will not be accepted.



The points $P(-3,0)$, $Q(-2,q)$, $R(5,6)$ and S are such that the perpendicular bisector of PR is QS, as shown in the diagram.

(i) Find the value of q. [4]

(ii) Show that the area of triangle PQR is 25 units^2. [2]

(iii) Find the coordinates of S, given that the area of the quadrilateral $PQRS$ is 75 units^2. [4]

(iv) Find the length of the perimeter of $PQRS$. [2]

7.

The lines $y - 3x = 1$ and $y + 2x = 6$ meet at the point A. Find

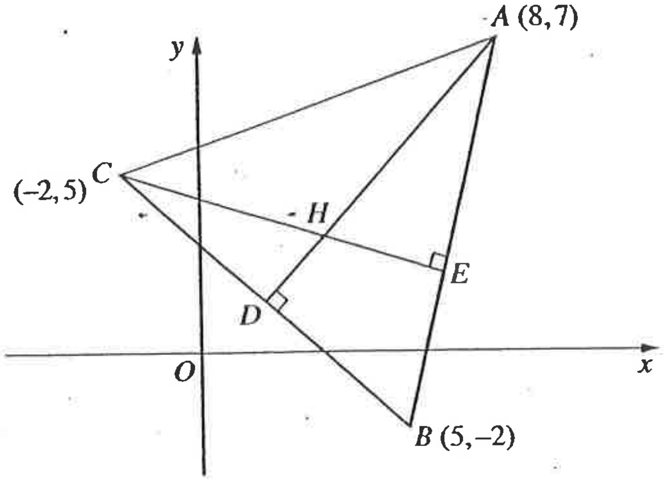
(i) the equation of the line through A which passes through the point B (3, 8),

(ii) the equation of the line through A which is perpendicular to AB.

[5]

COORDINATE GEOMETRY II

8. **Solutions to this question by accurate drawing will not be accepted.**



The vertices of the triangle ABC have coordinates (8, 7), (5, −2) and (−2, 5), as shown in the diagram. AD and CE are perpendicular to BC and AB respectively, and AD and CE meet at the point H. Find

(i) the coordinates of D and of H, [7]

(ii) the ratio $AD : HD$, [2]

(iii) the area of triangle ABC and of triangle HBC. [3]

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9. The points P(2, 6) and R(12, 1) lie on a circle at the ends of a diameter.

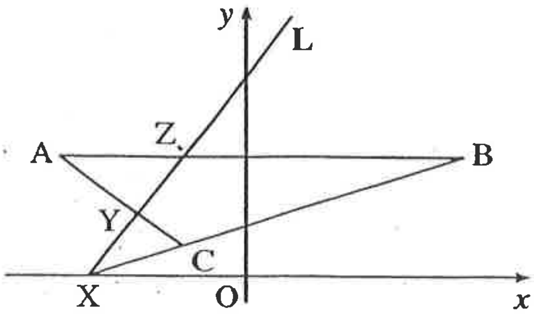
(i) Show that the circle has equation $x^2 + y^2 - 14x - 7y + 30 = 0$. [5]

(ii) Show that the point Q(6, 9) lies on the circle and that angle PQR = 90°. [2]

(iii) Find the coordinates of the point S, which lies on the circle and is such that PR and QS are perpendicular. [7]

(Turn Over)

10.



Three points A, B and C have coordinates (–6, 4), (7, 4) and (–2, 1) respectively. The point X lies on BC produced, as shown in the diagram, and BC : CX = 3 : 1.

(i) Show that the coordinates of X are (–5, 0). [3]

(ii) Find the equation of the line **L** which has gradient $\frac{4}{3}$ and passes through X. [2]

(iii) Calculate the ratio AZ : ZB, where Z is the point of intersection of AB with the line **L**. [4]

(iv) Show that AC is perpendicular to **L** and that CZ is perpendicular to AZ. [3]

(v) By using the similar triangles CZA and ZYA, or otherwise, calculate the length of AY, where Y is the point of intersection of AC with the line **L**. [2]

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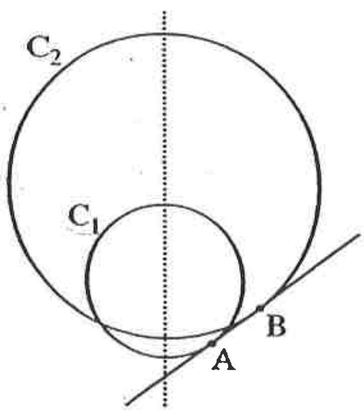
11. **(i)** The circle C_1 has equation $x^2 + y^2 + 18y - 319 = 0$. Show that it has radius 20 and find the coordinates of its centre. [3]

(ii) The circle C_2 has radius 40 and its centre is at (0, 16). Find the equation of C_2 in the form $x^2 + y^2 + ax + by + c = 0$. [3]

(iii) Verify that the point A(12, –25) lies on C_1 and that the point B(24, –16) lies on C_2. [2]

(iv) Calculate the gradient of the straight line through A and B. Hence, or otherwise, show that this line is the tangent to C_1 at A and also the tangent to C_2 at B. [4]

(v)



The diagram shows the circles C_1 and C_2 and the common tangent through A and B. Given that this common tangent has equation $4y - 3x + 136 = 0$, write down the equation of the other common tangent to C_1 and C_2. [2]